

# FEM modeling of five-phase magnetoelectric composites for energy transducers

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**A multiphysics FEM code is applied to model an energy transducer based on the five-phase magnetoelectric laminate composites. The formulation model includes the effect of the electrical load as well as the non-linearity of the thin-layer and bulk-layer magnetostrictive materials. This study provides a useful tool to study the influence of the thickness of the amorphous soft-magnetic alloy ribbons of five-phases ME transducers.**

*Index Terms*—FEM, multiphysics, piezoelectric, magnetostriction, magnetoelectric, energy transducer

## I. INTRODUCTION

THE research on smart multifunctional magnetoelectric (ME) composite materials has received significant attention in the applications such as the magnetic field sensors and energy transducers. In the case of laminated structures, such as the Terfenol-D/PZT/Terfenol-D for example, the ME composite exhibits a significant ME voltage coefficient (in V/Oe) according to different transversal (T) and longitudinal (L) magnetization–polarization modes under the resonance condition. In the LT-mode, when working as an energy transducer it can deliver an output power of few mW/cm<sup>3</sup> under the matching impedance condition. Nevertheless, in this case, a magnetostrictive material such as the Terfenol-D requires that the dynamic magnetic excitation field  $H_{ac}$  works around a significant magnetization bias  $H_{dc}$  to achieve a maximum ME sensitivity. Consequently, permanent magnets are usually needed that is an inconvenience for practical applications. To alleviate this limitation, recent studies on ME sensors propose five-phase hetero-structure ME composites (i.e. five layers) with the magnetization-graded ferromagnetic principle. In this way, the non-linear magnetic properties of bulk magnetostrictive layers such as the Terfenol-D or the Fe-Ni are combined with those of thin amorphous soft-magnetic alloy ribbons such as the Metglas (FeNiPB) or the FeCuNbSiB [1]. These ribbons have the benefit to exhibit a high permeability and a high piezomagnetic coefficient in low magnetization bias  $H_{dc}$ . Results have shown an enhancement of the sensor sensitivity for a magnetization bias  $H_{dc}$  close to zero. This improvement has been also observed for energy transducers with the experimentation [2]. However no numerical modelling is proposed in this study and the output deliverable power has not been addressed in this kind of transducer.

This paper proposes to use a 2D code based on the finite element method (FEM) to investigate five-phase laminated ME composites based on the magnetization-graded ferromagnetic principle. The formulation of the FEM code includes the shell element technique to take into account the thin layers [3] and takes into account the electrical effect when electrical impedance is connected between the electrodes of the piezoelectric layer. The nonlinear magnetic and

magnetostriction properties of the amorphous soft-magnetic alloy ribbons and bulk magnetostrictives layers are considered in the static regime by combining a quadratic model with the Jile-Atherton hysteresis model.

## II. FINITE ELEMENT FORMULATION OF THE FIELD PROBLEMS

The 2D FEM formulation of the coupled problem is derived by combining the magneto-electro-mechanical equilibrium equations 1-(a) and potential relations 1-(b) with the constitutive equations of magnetoelectric material (2). Using Einstein notation, they are, respectively,

$$\begin{cases} \sigma_{ij,j} = \rho_v \ddot{u}_i \\ D_{i,i} = 0 \\ \tau_{kij} e_k H_{j,i} = 0 \end{cases} \quad (a) \quad \begin{cases} \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \\ E_i = -V_{,i} \\ B_i = r^* a_{,i} \end{cases} \quad (b) \quad (1)$$

where  $\sigma_{ij}$ ,  $\varepsilon_{kl}$ ,  $u_i$  are the components of the mechanical stress tensor, mechanical strain tensor, the mechanical displacement.  $\rho_v$  is the mass density of the material.  $D_i$ ,  $E_i$ ,  $V$  are the displacement field, the electric field.  $B_i$ ,  $a$  and  $H_j$  are the induction magnetic field, the magnetic potential and the magnetic field.  $\tau_{kij}$  is the alternating tensor and  $e_k$  is the orthonormal basis.

$$\begin{cases} \sigma_{ij} = c_{ijkl}^E \varepsilon_{kl} + c_{ijkl}^B (\varepsilon_{kl} - \varepsilon_{kl}^\mu(B_k)) - e_{kij} E_k \\ D_i = -e_{ikl} \varepsilon_{kl} + \kappa_{ik} E_k \\ H_i = v_{ik}(B_k) B_k - h_{ikl}(B_k) \varepsilon_{kl} \end{cases} \quad (2)$$

where  $c_{ijkl}^E$  and  $c_{ijkl}^B$  are the strain tensors under constant electric and induction magnetic fields.  $\kappa_{ik}$ ,  $v_{ik}$  are, the elements of the permittivity matrix and reluctivity matrix.  $\varepsilon_{ij}^\mu(B_k)$  is the coercitive strain tensor due to magnetic force.  $e_{ikl}$  is the linear piezoelectric coefficients and  $h_{ikl}(B_k)$  is the relative nonlinear piezomagnetic coefficients defined as :

$$h_{ikl}(B_k) B_k = c_{ijkl} \varepsilon_{kl}^\mu(B_k) = \sigma_{kl}^\mu(B_k) \quad (3)$$

where  $\sigma_{ij}^\mu(B_k)$  represents the coercitive stress tensor due to magnetic force.

The finite element discretization yields the algebraic form (4) [3]:

$$[\mathcal{M}]\{\ddot{\mathcal{X}}\} + [\mathcal{C}]\{\dot{\mathcal{X}}\} + [\mathcal{K}]\{\mathcal{X}\} = \{\mathcal{F}\} \quad (4)$$

where  $[\mathcal{M}]$ ,  $[\mathcal{C}]$ ,  $[\mathcal{K}]$  represent respectively, the electro-magneto-mechanical mass, damping and stiffness global

matrices whereas  $\{\mathcal{F}\}$  represents the excitation vector. The unknowns of the problem in the vector  $\{X\}$  are the displacement  $\{u\}$ , the electric potential  $\{V\}$ , the magnetic vector potential  $\{a\}$  and the electrical charge  $\{Q\}$  in the electrodes of the piezoelectric layer when connected to an electrical load.

### III. MODELING OF THE NONLINEAR PROPERTIES

#### A. The nonlinear magnetic hysteresis model

The Jiles-Atherton hysteresis model for both the amorphous soft-magnetic alloy ribbons and the bulk magnetostrictive layers can be expressed as [5] :

$$H_e = H_i + \tilde{\alpha}M_i \quad (5)$$

with  $\tilde{\alpha} = \alpha + \alpha_M$ .  $M_i$  represents the total magnetization,  $\alpha$  is a dimensionless mean field parameter representing inter-domain coupling and  $\alpha_M$  a model parameter depending on the magnetic saturation and the mechanical prestress. Considering that the magnetostrictive material is subjected to an axially applied magnetic field, the parameter  $\alpha_M$  can be expressed [6] in ambient temperature as:

$$\alpha_M = \begin{cases} 2\sigma_o - 2\sigma_s \ln(\cosh(\sigma_o/\sigma_s))\lambda_s/\mu_o M_s^2 & \sigma_o/\sigma_s \geq 0 \\ 4\sigma_o - \sigma \ln(\cosh(\sigma/\sigma_s))\lambda_s/2\mu_o M_s^2 & \sigma_o/\sigma_s < 0 \end{cases} \quad (6)$$

where  $\lambda_s$  and  $M_s$  are, respectively, the maximum magnetostriction and the saturation magnetisation.  $\sigma_s$  is a reference stress and  $\sigma_o$  is the applied prestress.

The B-H curves from the Jile-Atherton hysteresis model are fitted and implemented in the FEM code with a polynomial equation.

#### B. The nonlinear magnetostriction model

Considering that the magnetic induction  $B_i$  is collinear to the easy magnetisation axe, the induced stress tensor force can be simplified by the following quadratic model [6]:

$$\varepsilon_{kl}^\mu = \frac{\beta_0}{2} (3B_k B_l - \delta_{kl} \|B_i\|^2) \quad (7)$$

where  $\beta_0$  is a fit magnetostriction coefficient and  $\|B_i\|$  is the norm of the magnetic induction field  $B_i$ . In this case, by combining the relations (3) and (6), the nonlinear piezomagnetic coefficients  $q_{ikl}(B_k) = h_{ikl}(B_k)v_{ik}^{-1}(B_k)$  in  $\text{Cm}^{-2}$  for magnetostrictive material can be expressed as:

$$q_{ikl}(B_k) = c_{ijkl}\varepsilon_{kl}^\mu(B_k)v_{ik}^{-1}(B_k) \quad (8)$$

#### C. Static and harmonic FEM resolutions

In harmonic analysis, the system (3) is solved with a linear model by considering a dynamic excitation field  $H_{ac}$  around a magnetization bias  $H_{dc}$ . The magnetostrictive coefficients  $q_{ikl}$  as well as the reluctivity  $v_{ik}$  are fixed according to the optimal magnetic  $H_{dc}$  bias point determined by resolving beforehand the nonlinear static FEM formulations. In this case, the nonlinear problem in static regime  $[\mathcal{K}]\{X\} = \{\mathcal{F}\}$  is solved with a piecewise-linear implementation of the constitutive law. The Newton-Raphson process methods is used to obtain the global incremental coefficients.

### IV. NUMERICAL MODEL

The FEM code has been applied to investigate the deliverable output power performances of the five-phase ME laminated composite represented in Fig.1. The hysteresis models are implemented in the FEM code for the Metglas (thin layer) and for the Terfenol-D (bulk layer). The obtained piezomagnetic coefficient  $d_{33}$  (in nm/A) (from  $d_{ikl} = c_{ijkl}^{-1}q_{ikl}$ ) of the Metglas and the Terfenol-D in function of bias field  $H_{dc}$  is shown in Fig.2.

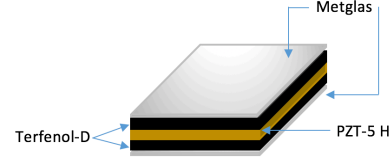


Fig. 1. The studied five-phase ME laminated composite

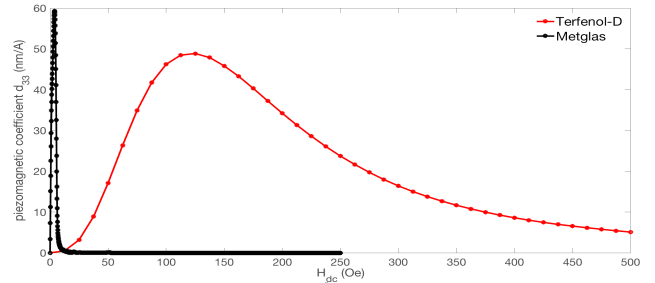


Fig. 2. The obtained piezomagnetic coefficient

### V. CONCLUSION

This paper proposes a FEM modeling to simulate a five-phase ME composite by combining a quadratic nonlinear magnetostriction stress tensor model and the Jile-Atherton hysteresis model. The detail of the procedure as well as the influence of the thickness of the amorphous soft-magnetic alloy ribbons on the ME voltage coefficient in static and dynamic regimes and on the output deliverable will be presented in the full version.

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